Interpretations of Probability

Preliminary Remarks

- Last week: Introduction to Probability and the Classical Interpretation

- Is probability an objective or a subjective notion?

- This week: Other Interpretations.

Subjective Interpretation

- Degree of belief/confidence/conviction/credence in some proposition.

  Problem: Research shows that most people (even experts) commit various fallacies that violate the probability axioms.

- A set of degrees of belief that violates the probability calculus is said to be *incoherent*.

  Solution: Focus on ideal rational person whose degrees of belief do not violate the probability calculus.

- Subjective probabilities $=_{df}$ coherent sets of degrees of belief.
- Prominent advocates: Bruno de Finetti, Frank Ramsey.

Subjective Interpretation (2)

- How do we elicit degrees of belief?
- By asking how much one would be willing to bet on a given belief coming true.
- One formulation goes like this:

  A's degree of belief in B is p (where $0 \leq p \leq 1$) if and only if A would buy or sell a bet at the price of p units of utility where 1 unit of utility is paid if B is the case, 0 if B is not the case.

- Eliciting degrees of belief through betting allows us to express the idea of coherent sets of degrees of belief in a rigorous way… via the Dutch book argument

Dutch Book

- A Dutch book is a set of bets that collectively guarantee the gambler's loss.
- Example:

  Suppose person A assigns the following probabilities to their beliefs: $p(X) = 0.55$ and $p(\sim X) = 0.55$ and the dealer accepts both bets from A, i.e. person A pays \$1.10.

  Scenario 1: X happens, so person A gets \$1. Net loss: \$0.10

  Scenario 2: X doesn't happen (i.e. ~X), A gets \$1. Net loss: \$0.10

  Conclusion: Person A loses either way.

- A person is subject to a Dutch book *if and only if* that person holds an incoherent set of degrees of belief (i.e. they violate the probability calculus).

Objections to Subjective

(1) Betting method problematic (refusal to bet, betting itself may alter belief, etc.)

Replies: Hypothetical scenario, low stakes, etc.

(2) Different people have different degrees of belief in the same propositions.

Reply: At least in the Bayesian case, there is a washing out/ swamping of the priors

(3) The only restriction imposed is coherence. This is too weak.

Reply: No probabilities in, no probabilities out.

Frequency Interpretation

- Some features:
  - Accords with our intuitions.
  - Employed widely in science (central in Statistics).
  - Probability an objective property of the world.
- Relative Frequencies: The proportion of favourable outcomes of attribute A in a reference class B: m(A)/n.
  - Finite Frequentism: Actual finite reference classes.
  - Infinite Frequentism: Infinite reference classes ($\lim_{n\to\infty} m(A)/n$)

  <u>Infinite reference classes/sequences</u>: We imagine a hypothetical infinite extension of an actual sequence of trials. The probability equals the limit of the sequence of relative frequencies.
- Prominent advocates: John Venn, Richard von Mises.

Objections to Frequency

(1) Extrapolating on the basis of a finite and limited portion of a sequence can be misleading.

Reply: Require a sufficiently large number of repetitions to minimise error.

(2) The problem of single-case probabilities.

Example: A coin tossed just once yields a relative frequency of heads equal to probability 1 or 0.

NB1: Many events are unrepeatable, e.g. events at the early history of the universe, the Queen's 1953 coronation, etc.

NB2: A coin that is never tossed (and thus yields no actual outcomes) lacks probability altogether.

Reply: Talk of probabilities outside repeatable cases does not make sense.

Objections to Frequency (2)

(3) Reference class problem: Relative frequencies are, by definition, relativised to a reference class. But, which one?

Example: What is the probability of Ioannis living to age 75? Qua male? Qua infrequent smoker? Qua philosophy lecturer? Qua XYZ? Qua all of these?

Problem 1: Reference class too narrow.

Problem 2: Reference class too wide.

Reply: The right reference class is determined by all the relevant evidence. What evidence is relevant is a problem for any account of probability.

Propensity Interpretation

- Probability $=_{df}$ the tendency of a system to produce a particular outcome.
- Probabilities are causal tendencies/dispositions/propensities of physical systems, i.e. they are objective features of the world.
- Especially useful in quantum physics where set-ups seem to be irreducibly indeterministic. Indeed, motivated by single-case probability attributions in quantum mechanics.

  Examples:

  (1) The probability of decay within the next hour is a physical property of a radioactive atom. (2) Biased die. The die itself has a propensity to produce a particular outcome, i.e. p(face 6) = 0.25.
- Prominent advocates: C.S. Peirce, K.R. Popper.

Objections to Propensity

(1) Not always clear whether the background knowledge contains enough information to allow inference of the causal tendency.

Reply: Employ frequencies to infer such knowledge where no other info is available.

(2) Propensity collapses to Frequentism.

Reply: Leave out talk of frequencies… "Probability is the propensity of a repeatable experimental set-up to produce sequences of outcomes".

(3) We don't always deal with irreducibly indeterministic scenarios. Indeed, some envisage a future science devoid of indeterministic scenarios.

Reply: Cannot dismiss these scenarios on a promissory note.

Logical Interpretation

- Interprets a conditional probability as a logical relation giving a degree of inductive support.

  P (H/E) is the degree of inductive support (partial entailment) that the evidence statement E gives to the hypothesis H.

- In this context, *logical probability, degree of confirmation* and *inductive probability* are synonymous.

  Keynes: Identifies *degree of partial entailment* with *degree of rational belief*.

- Prominent advocates: J.M. Keynes, R. Carnap.

Logical Interpretation (2)

- Measuring P(H/E) á la Carnap:

  State description: Complete description of a possible state of the universe.

  Example: In a universe with two objects (a, b) and one property (R) we can list all possible states of the universe as:
  1. Ra&Rb, 2. Ra&~Rb, 3. ~Ra&Rb, 4. ~Ra&~Rb.

  Range: The state descriptions corresponding to a given statement, e.g. (∃x) Rx corresponds to the disjunction 1v2v3.

  - We can now model deductive and inductive relationships

  Example: Evidence *Ra* rules out state descriptions 3 and 4.
  How does it support hypothesis H: (x) Rx?
  P(H/E) = range (H&E) / range (E) = 1/2.

Objections to Logical

(1) Learning from experience is impossible.

Suppose H: *Rb*. This holds in 2 state descriptions (1, 3). So the prior probability of the hypothesis is ½. If we now get evidence *Ra* (which holds in 1, 2), we have range (H&Ra overlap) / range (Ra) = 1/2.

Solution: Carnap proposed to evaluate the range of statements in a different way, viz. structure descriptions.

- Assigning unequal weights to state descriptions by identifying structure descriptions.

State Description	Weight	Structure Description	Weight
1. Ra&Rb	1/3	All R	1/3
2. Ra&~Rb	1/6	1 R, 1~R	1/3
3. ~Ra&Rb	1/6		
4. ~Ra&~Rb	1/3	No R	1/3

Objections to Logical (2)

- The weights are used as a measure of the ranges of statements:

  $c(H/E) = m\ (H\&E)/m(E)$

(2) How do we decide which measure to go for? Can we choose a unique non-arbitrary confirmation function?

NB: Carnap's only constraints are (1) the weights add up to one and (2) each state description has a weight greater than zero.

<u>Problem</u>:
Infinitely many confirmation functions satisfy these constraints.

Food for Thought

- Should we be pluralists with respect to the different interpretations of probability?

- If NO, which is the correct interpretation?

- If YES, do we accept all interpretations or just some? How do we decide which ones to apply when?

Reading

- Earman, J. and Salmon, W.C. (1999) 'The Confirmation of Scientific Hypotheses', in M. Salmon et al. (eds.) Introduction to the Philosophy of Science, Indianapolis: Hackett Publishing Company, ch. 2. pp. 77-89.