Critical Reasoning

‘Techniques in Reasoning’

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Modesty
Modesty

• Clearly, some claims are more modest than others.

Compare:
* The blues will win 2-0.

* The blues will win by a two goal margin.

Also:
* Smith stole the plans by breaking into the safe.

* Someone stole the plans by breaking into the safe.
Modesty and truth

• *More modest* claims have *more opportunities* to come out true.

**Compare:**
* The blues will win 2-0.

This comes out true only in the case where the game does end up 2-0.

* The blues will win by a two goal margin.

This comes out true in the following cases: 2-0, 3-1, 4-2, 5-3, 6-4, etc.
Modesty and truth (2)

• *More modest* claims have *more opportunities* to come out true.

**Compare:**
* Smith stole the plans by breaking into the safe.

This comes out true only in the case where Smith did indeed steal the plans by breaking into the safe.

* Someone stole the plans by breaking into the safe.

This comes out true in the following cases: Smith did, Jones did, Bates did, Dalton did, etc.
Modesty and defensibility: A rule

- On the assumption that we don’t already have any evidence, more modest claims are more defensible.
- That’s because a strictly greater number of pieces of evidence can support such claims.

<table>
<thead>
<tr>
<th>SMITH STOLE THE PLANS</th>
<th>SOMEONE STOLE THE PLANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVIDENCE THAT SMITH DID</td>
<td>EVIDENCE THAT SMITH DID</td>
</tr>
<tr>
<td>EVIDENCE THAT JONES DID</td>
<td>EVIDENCE THAT JONES DID</td>
</tr>
<tr>
<td>EVIDENCE THAT BATES DID</td>
<td>EVIDENCE THAT BATES DID</td>
</tr>
<tr>
<td>EVIDENCE THAT DALTON DID</td>
<td>EVIDENCE THAT DALTON DID</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

NB: Both claims could be false (e.g. if plans mislaid)
Modesty and defensibility

• Actually, there are some exceptions to this rule.

   Example:
   * The blues will win two trillion to nil.

   * The blues will win by a two trillion goal margin.

• Neither are practically possible. We can thus safely assume that they are both equally indefensible.

• That is, there are limits at which further modesty confers no additional defensibility.
• Given the above restrictions, we can reformulate the rule as follows:

If a claim is *at all* defensible, a more modest version will either *increase* or *maintain* its defensibility.
Which is more defensible? Joseph's visit will take place on

- Nov. 25th
- one of the following days: Nov. 24th, 25th or 26th
Give examples of pairs of claims where one is more defensible than the other.
Logical Strength
• Some claims are logically stronger than others.

A claim $X$ is *logically stronger* to a claim $Y$ if and only if one can derive/deduce $Y$ from $X$ but not $X$ from $Y$.

\[
\begin{align*}
X & \quad \checkmark \quad Y \\
\therefore Y & \quad \checkmark \quad \therefore X
\end{align*}
\]

• Whenever this asymmetry holds, we can say that the one claim $X$ is logically stronger than the other $Y$. 
Logical strength: Existential introduction

• We can diminish logical strength through the use of logical rules.

• **Existential Introduction Rule:**

\[ \text{Pa} \quad \text{John is ill} \]
\[ \implies \exists x \forall x \]
\[ \text{\therefore } \exists x \forall x \]
\[ \text{\therefore Someone is ill} \]

• Note that from ‘John is ill’ we can derive ‘Someone is ill’ but not vice-versa.

• After all, the latter claim may be made true by the fact that Jill is ill, Jane is ill, etc., but not John is ill.
• **Disjunction Introduction Rule:**

\[
P \quad \text{This coffee has cream.}
\]

\[
\therefore P \lor Q \quad \therefore \text{This coffee has cream or milk.}
\]

**NB:** It could be the case that this coffee has both.

• From ‘This coffee has cream’ we can derive ‘This coffee has cream or milk’ but not vice-versa.

• After all, the latter claim may be made true by the fact that the coffee has only milk and hence no cream.
• Conjunction Elimination Rule:

\[ P \land Q \quad \text{The winter will be cold and dry.} \]
\[ \therefore P \quad \therefore \text{The winter will be cold.} \]

• From ‘The winter will be cold and dry’ we can derive ‘The winter will be cold’ but not vice-versa.

• After all, the latter claim may be made true even if the winter is actually wet (and hence not dry).
Logical strength, modesty and defensibility

• We can now connect the two notions:

*Diminishing logical strength results in the formation of more modest and hence more defensible claims.*

• That’s because logically stronger claims commit themselves to more than logically weaker ones.

• In doing so, logically stronger claims have *less opportunities* to come out true.
• Knowing logical rules can help you win arguments!!!

**Example:**
If a successful counter is launched against a view, you *can* attempt to salvage it by making it more modest.

<table>
<thead>
<tr>
<th>Opponent’s View</th>
<th>Your View</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modest B’</td>
<td>Original B</td>
</tr>
</tbody>
</table>
Which one is logically stronger?

- Joker is a psycho and Batman is a narcissist and someone is a narcissist.
- Batman is a narcissist and Joker is a psycho and Batman is a narcissist.

Start the presentation to see live content. Still no live content? Install the app or get help at PollEv.com/app.
Share some examples of views you once held and the more modest views you replaced them with.
Revising Premises
Duhem, Quine and their namesake thesis

- Duhem’s Thesis (1906): Hypotheses cannot be tested in isolation as they don’t have consequences on their own.

- The web-of-belief metaphor (Quine 1951): All our beliefs are nestled in an interconnected web of support.

- Despite some differences, the first view is now widely known as the Duhem-Quine thesis.
Systems

• Since the units we test contain not just a central hypothesis but also a number of auxiliaries we can call these ‘systems’.

• The consequences of systems can be expressed in the usual deductive form.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   \[ n. \text{Auxiliary assumption } n-1 \]
   \[ \therefore \text{Observational consequence } 1 \]

• When a consequence is true we can say that the system is confirmed. When false, we can say it’s disconfirmed.
Suppose a given consequence is indeed false. Should we throw away the system?

Surely, we would want to try to revise it first. But can we infer which premise is to blame?

Duhem’s answer: No!

That’s actually a matter of logic. Recall that a T/F conclusion can be validly derived from one or more false premises.

Otherwise put, the content of a conclusion – here a false one – is included in the premises but we don’t know where.
The possibilities

- When an observational consequence is false we can only infer that \textit{at least} part of the system is at fault.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   \begin{itemize}
   \item \underline{n. Auxiliary assumption \textit{n-1}}
   \end{itemize}
   \begin{itemize}
   \item \underline{\therefore Observational consequence \textit{1} (False)}
   \end{itemize}
The possibilities

• When an observational consequence is false we can only infer that *at least* part of the system is at fault.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   \[ n. \text{Auxiliary assumption } n-1 \]
   \[ \therefore \text{Observational consequence } 1 \text{ (False)} \]
The possibilities

- When an observational consequence is false we can only infer that at least part of the system is at fault.

  1. Central hypothesis
  2. Auxiliary assumption 1
  ...
  n. Auxiliary assumption n-1
  ∴ Observational consequence 1 (False)
• When an observational consequence is false we can only infer that *at least* part of the system is at fault.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   $n$. Auxiliary assumption $n-1$
   $\therefore$ Observational consequence 1 (False)
• When an observational consequence is false we can only infer that *at least* part of the system is at fault.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   \[ n. \text{Auxiliary assumption } n-1 \]
   \[ \therefore \text{Observational consequence 1 (False)} \]
The possibilities

• When an observational consequence is false we can only infer that \textit{at least} part of the system is at fault.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   \( n \). Auxiliary assumption \( n-1 \)
\[ \therefore \text{Observational consequence 1 (False)} \]
The possibilities

When an observational consequence is false we can only infer that *at least* part of the system is at fault.

1. Central hypothesis
2. Auxiliary assumption 1
   ...
   n. Auxiliary assumption n-1

∴ Observational consequence 1 (False)
Replacement, not addition

- If a consequence $C$ of a system (of theories or beliefs) is false, it is impossible to rectify it by merely adding premises.

- That’s because classical logic is monotonic: One can never remove content or consequences by adding premises.

  \[
  \Gamma \vdash C \\
  \therefore \Gamma, A \not\vdash C
  \]

- So, to remove an offending consequence, one or more parts of the system must be replaced or removed!

**NB:** Replacement is equivalent to removal + addition.
Analogy
Analogical reasoning

- The use of such reasoning is widespread throughout the scientific and everyday domains.

**Examples:**

* Newton’s law of gravity and Coulomb’s law.

* The actual bridge being proposed and real or simulated models of it.

* Natural selection and artificial selection.
Analogies and abstraction

- Analogical reasoning depends crucially on abstraction.
- The ability to abstract away from the particulars allows us to see whether other systems fit the same mould.

\[
\begin{align*}
F_G &= G \times m_1 \times m_2 / r^2 \\
A &= B \times C_1 \times C_2 / D^2 \\
F_C &= k \times q_1 \times q_2 / r^2
\end{align*}
\]
Limits: The uninformative end

• The strength of the analogy depends on how much we had to abstract away in order to find commonalities.

• At one end, the analogy is completely uninformative as we abstract away so much that what’s left is trivial.

Example:

* Newton’s equation has several variables.

* Therefore, others will too!
At the other end, the analogy is as informative as it gets, when only one layer is abstracted away.

In the mathematical case, this can be expressed through the notion of isomorphism.

Two structures are isomorphic IFF there is a bijective mapping between their objects that preserves relations.
Logic and abstraction

• Logic, by its very nature, abstracts. Its rules are meant to operate on structure, regardless of content.

1. All penguins are birds.
2. Chilly Willy is a penguin.
\[ \therefore \text{Chilly Willy is a bird.} \]

1. All Fs are Gs.
2. \( a \) is an \( F \).
\[ \therefore \text{\( a \) is a} \ G. \]

• In this respect, it can help us peel away layers through abstraction.
The End