Formal Logic

Lecture 2: The Semantics of Propositional Logic (Part I)

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Formalisation

Formalisation: Rules of thumb

- To formalise (or symbolise) is to transform natural language sentences into the sentences of formal logic.

- Rules of thumb concerning the choice of sentence letters:

1. Use letters in the formal language that are mnemonic, i.e. they help recall the natural language sentence they stand for.

2. Use the same letters in the formal language whenever you want to express the same sentence in the natural language.

Donald has children and Donald does not have children.

C & ¬C   where C: Donald has children.

Negations (not, it's not the case, etc.)

Natural language sentence:	**Formalised:**
The sky is grey.	G
It's not the case that the sky is grey.	¬G
The sky **isn't/is not** grey.	¬G
It's not the case that the sky is not grey.	¬¬G
Hilary was elected president.	E
Hilary **wasn't/was not** elected president.	¬E
Jim washed the dishes.	W
Jim **didn't/did not** wash the dishes.	¬W
The score seems fair.	F
The score **doesn't/does not** seem fair.	¬F

Negation range matters

Formalised:	**Natural Language**:
¬(O&I)	It's not the case that Othello and Iago love Desdemona (not both but one or neither do)
¬O&¬I	Othello doesn't love Desdemona and Iago doesn't love her, i.e. neither does.
¬O&I	Othello doesn't love Desdemona but Iago does, i.e. only one (Iago) loves her.
O&¬I	Othello loves Desdemona but Iago doesn't, i.e. only one (Othello) loves her.
where	O: Othello loves Desdemona I: Iago loves Desdemona

Conjunctions (and, but, as did, etc.)

Natural language sentence:	**Formalised:**
The sky is grey **and** the sea is blue.	G&B
The sky is grey **and** the sea blue.	G&B
The sky is grey **but** the sea blue.	G&B
It's not the case that both the sky is grey **and** the sea blue.	¬(G&B)
John loves sports **and** Amy loves sports.	J&A
John **and** Amy both love sports.	J&A
Stuart wrote novels **and** Stuart wrote poetry.	N&P
Stuart wrote novels **and** poetry.	N&P
Sam wrote novels, **as did** Kim.	S&K

Disjunctions (or, either... or..., etc.)

Natural language sentence:	**Formalised:**
The sky is grey **or** the sea is blue.	$G \vee B$
The sky is grey **or** the sea blue.	$G \vee B$
Either the sky is grey **or** the sea blue.	$G \vee B$
It's not the case that **either** the sky is grey **or** the sea blue.	$\neg(G \vee B)$
Eve will finish the race **or** Jason will finish it.	$E \vee J$
Eve **or** Jason will finish the race.	$E \vee J$
Jim plays the banjo **or** Jim plays the piano.	$B \vee P$
Jim plays either the banjo **or** the piano.	$B \vee P$

Inclusive vs. exclusive disjunction

- **Inclusive disjunction:**

  Either Ann will graduate **or** Ben will graduate this year.

  This means at least one will graduate.

  By default, the disjunction symbol conveys this meaning: $A \vee B$

- **Exclusive disjunction:**

  Either Cid will win the 100m race **or else** David will win it.

  This means exactly one of them will win the 100m race.

  Formalised as: $(C \vee D) \& \neg(C \& D)$

  NB: In computer science/engineering, 'XOR' and '$\oplus$' are used to denote exclusive disjunctions.

Conditionals (if… then…, if, only if, provided, etc.)

Natural language sentence:	**Formalised:**
If the car will go, **then** the tank has fuel.	$C \rightarrow F$
If the car will go, the tank has fuel.	$C \rightarrow F$
The tank has fuel **if** the car will go.	$C \rightarrow F$
The car will go **only if** the tank has fuel.	$C \rightarrow F$
It's not the case that **if** the car will go, the tank has fuel.	$\neg(C \rightarrow F)$
Zoe will win **provided** she scores at least 5 points.	$S \rightarrow W$
Zoe will win **if** she scores at least 5 points.	$S \rightarrow W$
Sid will recover **so/as long as** he gets the medicine.	$M \rightarrow R$
Sid will recover **in case** he gets the medicine.	$M \rightarrow R$
Assuming Sid gets the medicine, he will recover.	$M \rightarrow R$

Bi-conditionals (if and only if, just in case, etc.)

Natural language sentence:	**Formalised:**
CO_2 emissions will be cut **if and only if** all countries take immediate action.	C↔I
CO_2 emissions will be cut **if but only if** all countries take immediate action.	C↔I
CO_2 emissions will be cut **just in case** all countries take immediate action.	C↔I
Alex will answer **all and only** the hard questions.	A↔Q
Alex will answer the questions **iff** they are hard.	A↔Q
If Jon gets a raise, Jill promoted him **and if** Jill promoted him, Jon gets a raise.	R↔P
Jon gets a raise **iff** Jill promoted him.	R↔P

Arguments (therefore, thus, so, hence, etc.)

Example:

1. **If** it rains, **then** the dam will burst.
2. It rained.

Therefore, the dam burst.

1. $R \rightarrow B$
2. R

$\therefore\ B$

where: R: It rains. B: The dam will burst.

NB: Terms like 'therefore', 'thus', 'so' and 'hence' indicate that the sentence after those terms is (meant to be) a logical consequence of the preceding sentences.

Tensed cases

- Wherever possible, we ignore tense differences by using the same letter in the formal language.

  Example: R stands for both ‘it rains’ and ‘it rained’.

- That these differences don’t matter in certain cases can be shown by fixing the natural language expressions.

  1. If it rains sufficiently on 06/01/20, then the dam bursts by 07/01/20.
  2. It rains sufficiently on 06/01/20.

  Therefore, the dam bursts by 07/01/20.

Ambiguous cases

- The meaning of some natural language sentences is ambiguous. L_1 allows for their disambiguation.

  Example:
  Bob teaches math **and** Ali teaches math **or** Sue teaches math.

  The above is ambiguous between:
  1. Either Bob and Ali teach math or Sue does.
  2. Bob teaches math and either Ali or Sue teach math.

  We can disambiguate via two formalisations:
  1a. (B&A)VS
  2a. B&(AVS)
  } logically inequivalent

Unambiguous cases

- Sentences that are exclusively conjunctions or exclusively disjunctions are not ambiguous.

Examples:
Bob teaches math and Ali teaches math and Sue teaches it.

B&(A&S) (B&A)&S	}	logically equivalent

Bob teaches math or Ali teaches math or Sue teaches it.

B∨(A∨S) (B∨A)∨S	}	logically equivalent

NB: For this reason, some logicians adopt the convention of dropping the brackets, e.g. B&A&S. We refrain from doing so!

Truth Tables: The Semantics of L_1

Truth-assignment

- For the time being, forget about what you know about semantics or meaning in the everyday usage of the term.

- In this context, we give meaning to sentences by **assigning truth-values** to each symbol denoting a proposition.

  NB: This is also known as **interpreting** the propositions.

- In classical logic, we have two such values: **T** (truth), **F** (false).

- Here's one such assignment/interpretation:

P	Q
T	F

Semantic compositionality

- Our language is semantically **compositional**:

  Compound sentence truth-values are calculated on the basis of atomic sentence truth-values plus logical connectives.

  Example:

P	Q	P&Q
T	F	T ? F

- For that we need to know how the logical connectives affect truth-values. That's what we consider next.

Truth-tables: Conjunction and disjunction

Conjunction

P	Q	P & Q		
T	T	T	T	T
T	F	T	F	F
F	T	F	F	T
F	F	F	F	F

Disjunction

P	Q	P v Q		
T	T	T	T	T
T	F	T	T	F
F	T	F	T	T
F	F	F	F	F

Truth-tables: Negation

Negation

P	¬P
T	FT
F	TF

Truth-tables: Conditional and bi-conditional

Conditional

P	Q	P → Q
T	T	T T T
T	F	T F F
F	T	F T T
F	F	F T F

Bi-conditional

P	Q	P ↔ Q
T	T	T T T
T	F	T F F
F	T	F F T
F	F	F T F

A conjunctive sentence is true just in case

at least one conjunct is true

both conjuncts are true

at least one conjunct is false

both conjuncts are false

A conditional is true just in case

both the antecedent and consequent are true

the antecedent is true and the consequent false

the antecedent is false or the consequent is true

both the antecedent and the consequent is false

The End