Formal Logic

Lecture 7: Formalisation in Predicate Logic

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Formalisation in Predicate Logic

Formalisation: Rules of thumb

- The same rules of thumb in choosing predicate letters apply here as those that applied in propositional logic:

  1. Use letters in L_2 that are mnemonic, i.e. they help recall the natural language predicates and constants they stand for.

  NB: The exception is variables; they neither stand for anything specific, nor do we have a choice (e.g. x, y and z).

  2. Use the same letters in L_2 whenever you want to express the same predicate, constant or variable.

  Boris has a cat and Boris does not have a cat.
  Cb & ¬Cb   where C: … has a cat, b: Boris.

Dictionaries

- Whenever we choose letters in L_2 to express predicates or designators in natural language, we are offering a *dictionary*:

Fj	where F: … is a fraudster
	j: John
Ie & Lf	where I: … is an island
	L: … is land-locked
	e: England
	f: France
Clts	where C: … is cheaper than … and …
	l: Lidl
	t: Tesco
	s: Sainsbury's

Splitting up predicates

- Sometimes there is a choice to be made between the use of *n* or *n*+*m* predicates where *m* > 0.

- For example, the natural language sentence:

  Buzz Aldrin is an American hero.

- Can be translated as EITHER:

  Ab where A: … is an American hero, b: Buzz Aldrin

- OR:

  Ab & Hb where A: … is an American, H:… is a hero
  b: Buzz Aldrin

The order of constants or variables

- As we have already seen, sometimes predicates in natural language relate two or more things together.

- Since the order in which this relation holds often matters we need a way to express this in L_2.

- This is done by ordering the appearance of constants or variables in the formulae and sentences of L_2.

- Compare:

Natural Language:	**Formalised:**
Tyson Fury beat Deontay Wilder.	Bfw
Deontay Wilder beat Tyson Fury.	Bwf

Formalisations (without quantifiers)

Natural language sentence:	**Formalised:**
Othello loves Desdemona.	Lod
The senate acquitted Trump.	Ast
Manchester is not between Dundee and Glasgow.	¬Bmdg
Big Bird is taller than Elmo and Oscar.	Tbeo
The queen gives parliament her speech.	Gqps
Tom hates Mary and Ebenezer is a scrooge.	Htm & Se
Mary doesn’t hate Tom and Ebenezer repents.	¬Hmt & Re
Jake spoils Ingrid or the sea is not calm.	Sji V ¬Cs
Ingrid spoils Jake if the sea is calm.	$Cs \rightarrow Sij$
Andy will fly if and only if the ticket is cheap.	$Fa \leftrightarrow Ct$
Po is a kung fu expert only if Shrek is blue.	$Ep \rightarrow Bs$
Neither Blake nor Lin eat seafood.	¬(Eb V El)

Personal pronouns

- Personal pronouns (e.g. I, you, her, she, him, he) refer back to some specific individual(s), so we can use the same constant.

Natural Language:	**Formalised**:
Ceasar came, **he** saw, **he** conquered.	Cc & (Sc & Qc)
Tim built the house but **he** didn’t pay for **it**.	Bth & ¬Pth
Ann will either sleep or **she** will sunbathe.	Sa V Ba
If Joe passes to Kim, **she** passes to Ed.	$Pjk \rightarrow Pke$

- It is not always possible to translate personal pronouns thus.

1. If a politician plays nice, **he** won’t be elected.
2. If a politician plays nice, **a politician** won’t be elected.

≠

NB: In 2, the politician is not necessarily the same one.

Quantifiers: Universal

- The universal quantifier ∀ binds with variables to express generalisations. The following are treated as the same:

  Every human is mortal.
  If something is human, it is mortal.
  For any x, if x is human, then x is mortal.

- We read '∀x' as 'For any x'. To express the above sentences, we need only add a conditional connective: $(\forall x)\ (Hx \rightarrow Mx)$.

Natural Language:	**Formalised**:
All Swiss people like tennis.	$(\forall x)\ (Sx \rightarrow Lxt)$
Not everything is gold.	$\neg(\forall z)\ Gz$
Each house is built to last.	$(\forall y)\ (Hy \rightarrow By)$
Any fool is capable of reading.	$(\forall x)\ (Fx \rightarrow Rx)$

Quantifiers: Existential

- The existential quantifier ∃ binds with variables to express a quantity of some things. The following are treated the same:

  Some humans are wise.
  There is at least one human who is wise.
  For at least one x, x is human and x is wise.

- We read '∃x' as 'For at least one x'. So, to express these sentences, we need only add a conjunction: (∃x) (Hx & Wx).

Natural Language:	**Formalised**:
Some spies are traitors.	(∃y) (Sy&Ty)
At least one cat has fluffy fur.	(∃z) (Cz&Fz)
There exist some ghost.	(∃x) Gx
There is at least one sad clown.	(∃y) (Cy&Sy)

Quantifiers and pronouns

- Let's go back to the two natural language sentences:

  1. If a politician plays nice, he won't be elected.
  2. If a politician plays nice, a politician won't be elected.

- How are these translated? Here's what they seem to say:

  The first: 'All politicians who play nice won't be elected'. $(\forall x)((Px \,\&\, Nx) \rightarrow \neg Ex)$

  The second: 'If some politician plays nice, then some politician won't be elected'. $(\exists x)(Px \,\&\, Nx) \rightarrow (\exists y)(Py \,\&\, \neg Ey)$

  It may also say: 'For all politicians who play nice, some politician won't be elected'. $(\forall x)((Px \,\&\, Nx) \rightarrow (\exists y)(Py \,\&\, \neg Ey))$

Quantifiers: The binary connectives

- Be careful how you combine quantifiers and connectives:

Compare:

All tigers are meat-eaters. $(\forall x)\ (Tx \rightarrow Mx)$

All things are meat-eating tigers. $(\forall x)\ (Tx\ \&\ Mx)$

All things are either tigers or meat-eaters. $(\forall x)\ (Tx \vee Mx)$

All things that are meat-eaters are tigers
and vice-versa. $(\forall x)\ (Tx \leftrightarrow Mx)$

There are some doctors who are surgeons. $(\exists y)\ (Dy\ \&\ Sy)$

Some things are such that they are either
not doctors or they are surgeons. $(\exists y)\ (Dy \rightarrow Sy)$

There are some things that are either doctors or surgeons.
$(\exists y)\ (Dy \vee Sy)$

There are some things that are doctors just in case they are
surgeons. $(\exists y)\ (Dy \leftrightarrow Sy)$

Quantifiers: The unary connective

- The unary connective (i.e. negation) together with the quantifiers allow us to express the following possibilities:

Everything has mass.	$(\forall x)\ Mx$
Not everything has mass.	$\neg(\forall x)\ Mx$
Everything doesn't have mass.	$(\forall x)\ \neg Mx$
Something has mass.	$(\exists x)\ Mx$
Nothing has mass.	$\neg(\exists x)\ Mx$
Something doesn't have mass.	$(\exists x)\ \neg Mx$

- Note that some of the above have the same meaning. In fact, we can express the relationship between them as follows:

$\neg(\forall x)\ \phi x$ if and only if $(\exists x)\ \neg\phi x$

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Quantifiers: Other equivalences

- The following (same-coloured) pairs are also equivalent:

Everything with mass is extended.	$(\forall x)(Mx \rightarrow Ex)$
Nothing with mass is not extended.	$\neg(\exists x)(Mx \,\&\, \neg Ex)$
Something with mass is extended.	$(\exists x)(Mx \,\&\, Ex)$
Not everything with mass is not extended.	$\neg(\forall x)(Mx \rightarrow \neg Ex)$
Not everything with mass is extended.	$\neg(\forall x)(Mx \rightarrow Ex)$
Something with mass is not extended.	$(\exists x)(Mx \,\&\, \neg Ex)$
Everything with mass is not extended.	$(\forall x)(Mx \rightarrow \neg Ex)$
Nothing with mass is extended.	$\neg(\exists x)(Mx \,\&\, Ex)$

Quantifiers: Other equivalences (2)

- Here are some more possibilities:

Everything without mass is extended. $(\forall x)\ (\neg Mx \rightarrow Ex)$

Everything is such that it has mass and it is not extended. $(\forall x)\ \neg(Mx \rightarrow Ex)$

Something without mass is extended. $(\exists x)\ (\neg Mx \ \&\ Ex)$

There's something that neither has mass nor is it extended. $(\exists x)\ \neg(Mx \ \&\ Ex)$

Multiple quantifiers

- Sometimes it may not be easy to see that two or more quantifiers are needed to formalise a sentence.

- Take the natural language sentence:

  Every student has a computer.

- One might be tempted to formalise this sentence as follows:

  $(\forall x)\ (Sx \rightarrow Hxc)$

- This is incorrect for a number of reasons. We need an existential quantifier.

  $(\forall x)\ (Sx \rightarrow (\exists y)\ (Hxy\ \&\ Cy))$

Multiple quantifiers (2)

- Take the natural language sentence:

  If the weather is stormy, then Katherine drinks a hot chocolate or a tea.

- We can translate this as follows:

  $Sa \rightarrow (\exists y)\,(Dky\;\&\;(Cy \vee Ty))$

  where S: ... is stormy
  D: ... drinks ...
  C: ... is hot chocolate
  T: ... is tea
  k: Katherine
  a: weather

Multiple quantifiers (3)

- Take the natural language sentence:

  There is a city between London and Edinburgh.

- We can translate this as follows:

  $(\exists z)\ (Cz\ \&\ Bzle)$

  where C: ... is a city
B: ... is between ... and ...
l: London
e: Edinburgh

Multiple quantifiers and their order

- Sometimes we have multiple quantifiers overlapping in scope.

Natural Language:	**Formalised**:
Everyone loves everyone.	$(\forall x)(\forall y)\ Lxy$
Everyone loves at least one person.	$(\forall x)(\exists y)\ Lxy$
There is a person who loves everyone.	$(\exists x)(\forall y)\ Lxy$
No person loves everyone.	$\neg(\exists x)(\forall y)\ Lxy$
No person loves their self.	$\neg(\exists x)Lxx$

- 'Everyone loves someone' is ambiguous in natural language. We can resolve this ambiguity via the order of the quantifiers.

For any person, there's someone they love.	$(\forall x)(\exists y)\ Lxy$
Someone – same one(s) – is loved by everyone.	$(\exists y)(\forall x)\ Lxy$

The End